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Abstract

This paper derives a new criterion for the determination of the number of factors in static approximate factor models, that is strongly associated with the scree test. Our criterion looks for the number of eigenvalues for which the difference between adjacent eigenvalue-component number blocks is maximized. Monte Carlo experiments compare the properties of our criterion to the Edge Distribution (ED) estimator of Onatski (2010) and the two eigenvalue ratio estimators of Ahn and Horenstein (2013). Our criterion outperforms the latter two for all sample sizes and the ED estimator of Onatski (2010) for samples up to 300 variables/observations.

Keywords: static factor model, number of factors, test

JEL-code: C32, C52, C82

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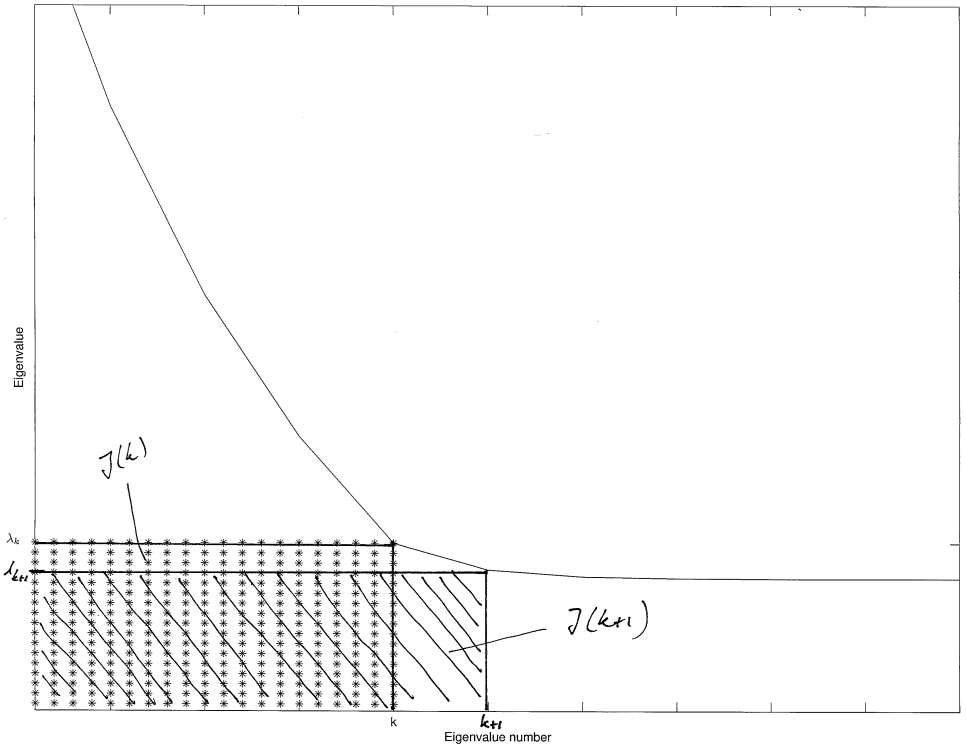
1 Introduction

A widely used method to analyse large quantities of data in the social sciences is factor analysis, in which the variation in a large number of observed variables is described in fewer unobserved variables, or movements in a large number of series are driven by a limited set of common 'factors'. One of the issues in factor analysis is the determination of the number of unobserved variables to retain, i.e. the number of factors. Various methods are in use: (i) heuristic methods like the Kaiser criterion in which only factors with eigenvalues greater than 1 are retained, or the scree test of Cattell (1966), which will be explained in more detail below; (ii) stopping rules, see e.g. Peres-Neto, Jackson and Somers (2005); or (iii) principal components analysis, see e.g. Jolliffe (2002, Chapter 6) or Coste *et al.*, (2005).

In recent years, large dimensional factor models have become more and more popular in econometric research too. For an overview of recent developments see Bai and Ng (2008) or Stock and Watson (2011). The determination of the number of factors is still high on the research agenda despite the fact that many studies have proposed solutions and consistent estimators using different factor model and distributional assumptions. Connor and Korajczyk (1993), Bai and Ng (2002), Onatski (2009), Onatski (2010), Ahn and Horenstein (2013) and Harding (2013) develop estimation methods for static factor models. Rcent examples for dynamic factors are Amengual and Watson (2007), Hallin and Liška (2007), Bai and Ng (2007), Jacobs and Otter (2008), Kapetanios (2010) and Breitung and Pigorsch (2013).

Figure 1: Graphical illustration of our criterion in a scree plot



We derive a criterion for the determination of the number of factors in approximate static factor models, that is strongly associated to the scree test. This is a graphical technique, which consists of plotting the eigenvalues λ_k against its component number, and deciding at which value of k the slopes of the plotted points are 'steep' to the left of k and 'not steep' to the right of k. This value of k, which defines an 'elbow' in the graph, is then taken to be the number of factors to be retained. Our criterion is based on the comparison of surfaces under the scree plot, as illustrated in Figure 1. We look for the value of k for which the difference between the adjacent products of the component numbers times the corresponding eigenvalue, in

other words the difference between adjacent eigenvalue-component number blocks ($J(k) - J(k+1) \equiv k\lambda_k - (k+1)\lambda_{k+1}$), is maximized.

In simulation experiments we compare our criterion to a couple of other estimators based on eigenvalues and also associated to the scree test. The Edge Distribution (ED) estimator of Onatski (2010) is based on the fact that any finite number of the largest of the bounded eigenvalues of the sample covariance matrix cluster around a single point. His estimator consistently separates the diverging eigenvalues from the cluster and counts the number of the separated eigenvalues, which is his estimate of the number of factors. Ahn and Horenstein (2013) propose the Eigenvalue Ratio (ER) and the Growth Ratio (GR) estimators. The ER estimator is obtained by maximizing the ratio of two adjacent eigenvalues arranged in descending order

$$\frac{\lambda_k}{\lambda_{k+1}} = \frac{V\left(k-1\right) - V\left(k\right)}{V\left(k\right) - V\left(k+1\right)}$$

while the GR estimator maximizes

$$\frac{\ln\left(V\left(k-1\right)\right) - \ln\left(V\left(k\right)\right)}{\ln\left(V\left(k\right)\right) - \ln\left(V\left(k+1\right)\right)} = \frac{\ln\left(1+\lambda_k^*\right)}{\ln\left(1+\lambda_{k+1}^*\right)}$$

where $V(k) = \sum_{j=k+1}^{m} \lambda_j$, $\lambda_k^* \equiv \frac{\lambda_k}{\sum_{j=k+1}^{m} \lambda_j}$ and $m = \min(n, T)$ for the number of variables n and the number of observations T. Our criterion outperforms the two eigenvalue test ratios of Ahn and Horenstein (2013) for all sample sizes. It also outperforms the ED estimator of Onatski (2010) for samples up to 300 variables/observations. This conclusion is robust for variation in the

signal to noise ratio and situations where "weak" factors are present, which may have a huge impact (Onatski 2012).

The rest of the paper is structured as follows. Section 2 describes our criterion and shows its consistency using the set-up of Onatski (2010). Section 3 presents Monte Carlo simulation experiments. Section 4 concludes.

2 Method

Our criterion

Consider the factor model

$$\boldsymbol{x}_t = \boldsymbol{\Lambda} \boldsymbol{f}_t + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\boldsymbol{x}_t$ is an n-vector of variables, $\boldsymbol{\Lambda} \equiv (\lambda_1 \ \ldots \lambda_n \ )'$, $\lambda_i \in R^k$, and the factors $\boldsymbol{f}_t \in R^k$ and the idiosyncratic components $\boldsymbol{\varepsilon}_t$ are independent. Let $\boldsymbol{x}_t$ be a $(n \times 1)$ stochastic normalized vector with zero mean and (stationary) covariance matrix $\mathrm{E}\{\boldsymbol{x}_t \boldsymbol{x}_t'\} = \boldsymbol{V} = \boldsymbol{C}\boldsymbol{\Lambda}\boldsymbol{C}'$ with ordered eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. Suppose $\boldsymbol{x}$ can be explained by a factor model, then we can decompose the Frobenius norm of matrix $\boldsymbol{x}$ as

$$\{||\boldsymbol{x}_t||^2\} = \mathrm{tr}(\mathrm{E}\{\boldsymbol{x}_t \boldsymbol{x}_t'\}) = \sum_{j=1}^{n} \lambda_j = \sum_{j=1}^{k} \lambda_j + \sum_{j=k+1}^{n} \lambda_j = n, \tag{2}$$

where $\sum_{j=1}^{k} \lambda_j$ is the explained variance using a factor model with rank k.

The *minimum* explained variance is $J(k) \equiv k\lambda_k$, because $\sum_{j=1}^{k} \lambda_j = k\lambda_k + \sum_{j=1}^{k} (\lambda_j - \lambda_k)$. The variance of $\boldsymbol{x}_t = J(k) + J^c(k)$, where $J^c(k) = \sum_{j=1}^{k} (\lambda_j - \lambda_k) + S_E$, with $S_E = \sum_{j=k+1}^{n} \lambda_j$ being the residual variance. The

aim is to maximize the minimum of $J(k)$ or to minimize the maximum of $J^c(k)$, where $\sum_{j=1}^{k}(\lambda_j - \lambda_k)$ is the penalty function. Whenever $J(k)$ has some maximum $k = r$, $DJ(k) = J(k) - J(k+1) = k(\lambda_k - \lambda_{k+1}) - \lambda_{k+1}$ is smaller than zero for $k < r$ and larger than zero $k \geq r$, implying $DJ(r-1) < 0$ and $DJ(r) > 0$. In the simulations below in Section 3, we employ the DJS estimator which consists of the contrained optimization $\widehat{k} = \underset{k}{\text{argmax}}\ \widehat{DJ}(k)$ with the constraint that $\widehat{DJ}\left(\widehat{k} - 1\right) < 0$. Since this constraint does not affect the asymptotic properties of our criterion, we will investigate the asymptotics of $DJ(k)$ using the model of Onatski (2010).

Asymptotics

Onatski (2010) considers the approximate factor model

$$\boldsymbol{X}^{(n,T)} = \boldsymbol{\Lambda}^{(n,T)}\boldsymbol{F}^{(n,T)} + \boldsymbol{e}^{(n,T)}, \tag{3}$$

where $\boldsymbol{X}^{(n,T)}$ is an $n \times T$ matrix of data on n cross-sectional units observed over T time periods, $\boldsymbol{\Lambda}^{(n,T)}$ is an $n \times r$ matrix whose (i,j)th element is interpreted as the loading of the jth factor on the ith cross-sectional unit, $\boldsymbol{F}^{(n,T)}$ is an $r \times T$ matrix whose (j,t)th element is interpreted as the value of the jth factor at time T, and $\boldsymbol{e}^{(n,T)} = \boldsymbol{A}\boldsymbol{\varepsilon}\boldsymbol{B}$ is an $n \times T$ matrix of the idiosyncratic components of the data, and $\boldsymbol{A}$ and $\boldsymbol{B}$ are two largely unrestricted deterministic matrices, and $\boldsymbol{\varepsilon}$ is an $n \times T$ matrix with i.i.d. Gaussian entries, so that both the cross-sectional and temporal correlation of the idiosyncratic terms are allowed.

Let the ordered eigenvalues of the sample matrix $\left(\boldsymbol{X}^{(n,T)\prime}\boldsymbol{X}^{(n,T)}\right)/T(n)$ be $\lambda_1^{(n,T)} \geq \lambda_2^{(n,T)} \geq \ldots \geq \lambda_m^{(n,T)}$ with $m = \min(n,T)$ and $\sum_{i=1}^{m} \lambda_i^{(n,T)} = n$. We assume that Assumption 1 and 2, Lemma 1–3, and Theorem 1 of Onatski (2010) hold.

Let $n/T(n) \to c > 0$ as $n \to \infty$. Let $k_{max}/n \to 0$ as $n \to \infty$ with the maximum possible number of factors k_{max} assumed a priori given sample size n, $T(n)$. Let $\widehat{DJ}(k) = k\left(\widehat{\lambda}_k^{(n,T)} - \widehat{\lambda}_{k+1}^{(n,T)}\right) - \widehat{\lambda}_{k+1}^{(n,T)}$. Our estimator $\widehat{DJ}(k)$ fits in the family of estimators of Onatski (2010, Equation (10))

$$\hat{r}(\hat{\delta}_{k+1}) = \max\left\{k \leq k_{max} : \ \hat{\lambda}_k^{(n,T)} - \hat{\lambda}_{k+1}^{(n,T)} \geq \hat{\delta}_{k+1}\right\}, \tag{4}$$

with $\hat{\delta}_{k+1} = \frac{n}{k}\hat{\lambda}_{k+1}^{(n,T)}$. So, one way of bounding the constant δ given k_{max} would be $\hat{\delta} = \frac{n}{k_{\max}}\hat{\lambda}_{k_{max}+1}^{(n,T)}$.

Onatski (2010, p1007) writes that his Theorem 1 suggests a way to estimate r. For $k(n) > r$ and large enough n, $\hat{\lambda}_{k+1}^{(n,T)}$ and hence $\hat{\delta}_{k+1}$ is finite with probability one as $n \to \infty$. For any $k \geq r$ the difference $\left(\hat{\lambda}_k - \hat{\lambda}_{k+1}\right)$ converges to zero with probability one, while the difference $\left(\hat{\lambda}_r - \hat{\lambda}_{r+1}\right)$ diverges to infinity. So $\hat{r}(\hat{\delta}_{k+1}) \to r$ in probability as $n \to \infty$. It follows from Equation (4) that the estimator $\widehat{DJ}(k) = k\widehat{\lambda}_k^{(n,T)} - (k+1)\widehat{\lambda}_{k+1}^{(n,T)}$ converges as $n, T(n) \to \infty$ for $k > r$ while it diverges to infinity for $k = r$. Our threshold is equal to $\hat{\delta}_{k+1}$, which diverges to infinity for $k \leq r-1$, but is finite for $k \geq r$, as illustrated in Figure 4 below.

Remark 1. Onatski's family of estimators in Equation (4) is consistent even for weak factors, which are defined as factors whose explanatory power

for response variables grows slower than the rate of n. If the k-th factor is weak, then $\text{plim}_{m\to\infty} \lambda_k = 0$, but $\text{plim}_{m\to\infty} m\lambda_k = \infty$.

Remark 2. The eigenvalue ratio estimators of Ahn and Horenstein (2013) require more strict assumptions to prevent the denominator of the ratios from becoming equal to zero.

3 Monte Carlo experiments

We compare finite-sample simulations of our DJS estimator with the two alternatives proposed by Ahn and Horenstein (2013), the eigenvalue ratios ER and GR, and the ED estimator proposed by Onatski (2010). For all the estimators considered, the argument search is performed over $k = 1, \ldots, k_{max}$ with $k_{max} = 8$.

Along the lines of Bai and Ng (2002) and Onatski (2010), we employ the data generating process as specified in Ahn and Horenstein (2013). The foundation of the simulation exercise is the following approximate factor model:

$$x_{it} = \sum_{j=1}^{r} \lambda_{ij} f_{jt} + \sqrt{\theta} u_{it}; \qquad u_{it} = \sqrt{\frac{1-\rho^2}{1+2J\beta^2}} e_{it}, \tag{5}$$

where $e_{it} = \rho e_{i,t-1} + (1-\beta)\,\nu_{it} + \beta \sum_{h=\max(i-J,1)}^{\min(i+J,n)} \nu_{ht}$ and the ν_{ht} and λ_{ij} are all drawn from $N(0,1)$. The idiosyncratic components u_{it} are normalized such that their variances are equal to one for most of the cross-section units.[1] The control parameter θ is the inverse of the signal to noise ratio (SNR) for the individual factors because $\text{var}(f_{jt}) / \text{var}\left(\sqrt{\theta} u_{it}\right) = 1/\theta$. When it is necessary

[1] More specifically for units $J+1 \leq i \leq n-j$.

to change the SNRs of all factors, we adjust parameter θ. However, we also simulate a single weak factor by drawing from $N(0, \theta_{wf})$ for the weak factor and $N(0,1)$ for the other factors, so θ_{wf} represents the relative dominance (or weakness) of the single factor. The magnitude of the time series correlation in the idiosyncratic component is controlled by parameter ρ. Note that equation (5) describes the approximate static factor model, so no autocorrelation for the factors is assumed. Parameter β governs the magnitude of cross-sectional correlation and parameter J the number of correlated units. We will focus on the specification with both serially and cross-sectionally correlated errors, $\rho = 0.5$, $\beta = 0.2$, $J = \max(10, n/20)$. Although the means of the factors, the factor loadings and the idiosyncratic component are all zero in the data generating process (5), we use double demeaned data, i.e. $x_{it} - T^{-1}\sum_t x_{it} - n^{-1}\sum_i x_{it} + (nT)^{-1}\sum_{i,t} x_{it}$, in order to avoid the one-factor bias problem as identified by Brown (1989).[2]

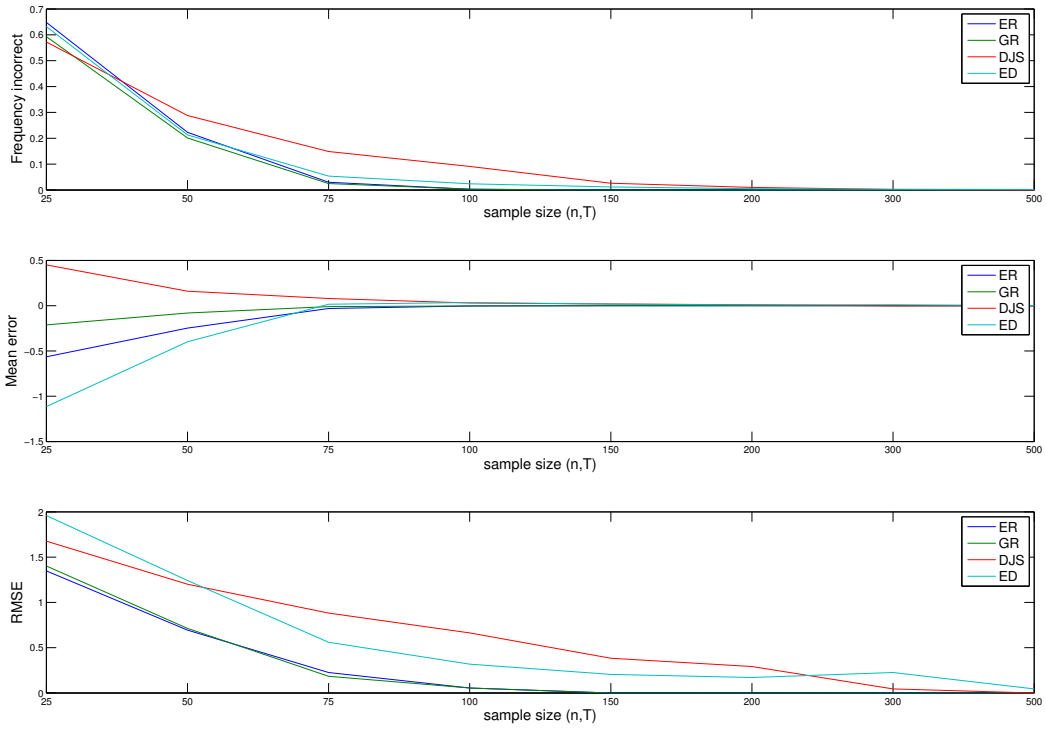
Base scenario

We focus on the model with $r = 3$ factors and configurations of the sample size over the grid $(n, T) = 25, 50, 75, 100, 150, 200, 300, 500$, inverse signal to noise parameter θ and the relative weakness of one of the three factors θ_{wf}. Based on 1000 simulations for each configuration, we compute the estimated number of factors $\widehat{k}$, i.e. the model and three performance statictis, the mean error, the root mean squared error (RMSE) and the frequency of the incorrect estimated number of factors, for each of the four different estimators DJS,

[2]We employ double demeaned data for the estimators of Ahn and Horenstein (2013) ER, GR and our proposed estimators DJS, but not for Onatski's (2010) ED estimator.

ER, GR and ED. In the simulations based on Ahn and Horenstein's (2013) baseline specification consisting of a three-factor model with $\theta_{wf} = \theta = 1$, the mode is equal to three factors for all estimators. Figure 2 shows the results for the performance statistics. The figure shows that our proposed estimator compares reasonably well with the other ones for this specification, although our DJS comes out less well than the others. The figure also shows that the AH estimators come out well in this base scneario with $\theta = \theta_{wf} = 1$. Below we will see that this conlusion is not robust.

Figure 2: Performance of different estimators



Note. Simulations are based on $\theta = \theta_{wf} = 1$.

Robustness

To check the robustness of the performance of the different estimators, we extend the grid for the inverse signal to noise parameter θ to 18 points: $\theta = [\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, 2, \frac{9}{4}, \frac{10}{4}, \frac{11}{4}, 3, \frac{13}{4}, \frac{14}{4}, \frac{15}{4}, 4, 5, 7, 12]$. In addition, we extend the grid for the weak factor parameter θ_{wf} to 15 points: $\theta_{wf} = \left[\frac{3}{32}, \frac{1}{8}, \frac{3}{16}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, 3, 4, 6, 8, 12\right]$. The combined grid consists then of $18 \times 15 = 270$ different configurations, each consisting of 1000 simulations. The performance of the different estimators for each configuration is summarized by the estimated number of factors, i.e. the mode of the simulations, and the frequency of the incorrect estimated number of factors. For each of the 270 configurations these performance statistics are further summarized by the percentages in Table 1. These represent the fraction of the different configurations for which the mode consists of the true number of factors, i.e. $\widehat{k} = 3$. The percentages presented in Table 2 consist of the fraction of the different configurations for which the frequency of incorrect estimated number of factors is larger than 10%. To keep the tables and outcomes readable, we only report outcomes for sample sizes where the number of variables n is equal to the number of observations T, as in Ahn and Horenstein (2013).

The first block in the two tables summarize the results of the 270 different configurations. The estimators DJS and ED clearly outperform the other alternatives for large sample sizes. While not visible in the standard configuration $\theta = \theta_{wf} = 1$ shown in Figure 2, the outperformance must by construction be due to variation in θ and θ_{wf}.

Table 1: Correct number of factors according to the different estimators

(n,T)	25	50	75	100	150	200	300	500
	$\forall\theta,\forall\theta_{wf}$, grid = 270							
ER	7	13	17	20	20	21	31	42
GR	5	10	14	16	17	19	28	38
DJS	27	33	37	40	37	39	50	56
ED	11	19	24	29	29	33	55	70
	$\forall\theta,\theta_{wf}=1$, grid = 18							
ER	22	44	50	61	67	72	89	94
GR	17	39	50	61	67	72	89	94
DJS	39	44	50	61	56	61	83	89
ED	17	39	44	56	56	61	89	94
	$\forall\theta,\theta_{wf}=[4,6,8,12]$, grid = 72							
ER	3	6	10	11	11	13	19	33
GR	0	1	4	4	4	4	10	21
DJS	51	60	68	71	68	71	85	94
ED	17	26	33	39	40	46	78	94
	$\forall\theta_{wf},\theta=1$, grid = 15							
ER	27	33	47	47	47	47	60	73
GR	27	27	40	40	40	40	47	67
DJS	60	60	60	60	60	60	60	60
ED	60	60	67	67	67	73	80	87
	$\forall\theta_{wf},\forall\theta>1$, grid = 225							
ER	1	6	9	12	12	13	24	36
GR	0	4	8	10	11	12	22	32
DJS	22	28	33	36	33	35	48	55
ED	0	9	15	20	20	24	49	65

Notes.
The results are based on the mode of 1000 Monte Carlo replications. The presented percentage is the fraction of the grid for which the mode equals the true number of factors $r = 3$ according to the different estimators. In case $\forall\theta,\forall\theta_{wf}$, the grid consists of $18\times15 = 270$ different simulations. The grid sizes for each case are reported on the first line.

Table 2: Frequency of incorrect number of factors according to the different estimators

(n, T)	25	50	75	100	150	200	300	500
	$\forall\theta, \forall\theta_{wf}, \text{grid} = 270$							
ER	100	97	93	90	89	87	77	62
GR	100	98	96	93	91	90	80	67
DJS	100	100	100	98	86	74	56	47
ED	100	96	89	83	81	77	56	35
	$\forall\theta, \theta_{wf} = 1, \text{grid} = 18$							
ER	100	89	78	72	67	61	28	11
GR	100	89	83	72	67	61	28	11
DJS	100	100	100	72	67	61	28	11
ED	100	94	78	72	61	56	17	6
	$\forall\theta, \theta_{wf} = [4, 6, 8, 12], \text{grid} = 72$							
ER	100	99	97	94	94	92	86	74
GR	100	100	100	99	99	99	93	83
DJS	100	100	100	100	83	53	21	11
ED	100	92	83	75	72	67	36	11
	$\forall\theta_{wf}, \theta = 1, \text{grid} = 15$							
ER	100	100	73	67	67	67	47	27
GR	100	100	87	73	73	73	53	40
DJS	100	100	100	93	60	40	40	40
ED	100	100	40	40	40	33	27	13
	$\forall\theta_{wf}, \forall\theta > 1, \text{grid} = 225$							
ER	100	100	99	97	96	95	85	70
GR	100	100	100	98	97	96	87	73
DJS	100	100	100	99	90	81	59	48
ED	100	100	99	93	91	87	63	40

Notes.
The results are based on the frequency of incorrect estimated number of factors for 1000 Monte Carlo replications. The presented percentage is the fraction of the grid for which the frequency of incorrectly estimated number of factors is larger than 10%. In the case $\forall\theta, \forall\theta_{wf}$, the grid consists of $18 \times 15 = 270$ different simulations. Grid sizes are denoted by 'grid'.

The second block in the two tables only considers the configurations without weak factors, i.e. $\theta_{wf} = 1$, and summarizes the results based on the 18 different configurations for θ. In the absence of weak factors no obvious differences exist in the performance of the proposed estimators, except perhaps for very small sample sizes. All the estimators are equally robust to variations in the signal to noise ratio.

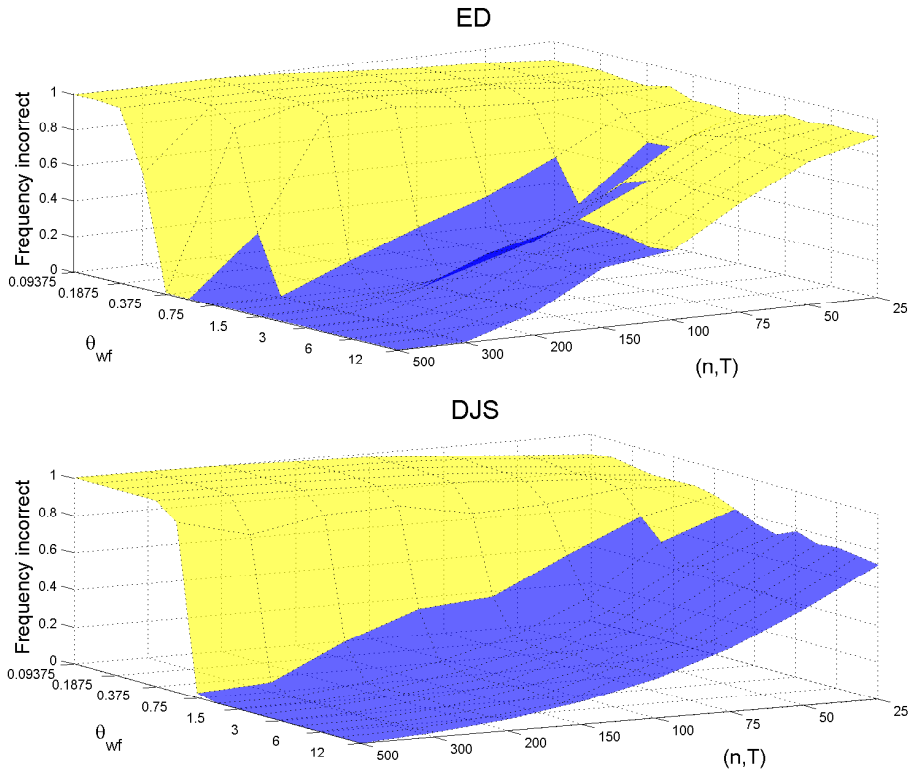
The third panel in Table 1 and Table 2 summarizes the results over the grid $\theta_{wf} = [4, 6, 8, 12]$ and clearly shows that the estimators ER and GR are not robust to the presence of weak factors. Moreover, the tables reveal that for the smaller sample sizes, the mode for DJS more often correctly estimates the true number of factors than ED, while ED shows a lower frequency of incorrect estimated number of factors.

The fourth panel in Table 1 and Table 2 considers the baseline case for the signal to noise ratio, i.e. $\theta = 1$, and summarizes the results based on the 15 different weak factor configurations θ_{wf}. In these configurations, the ER and GR estimators perform quite comparably to the alternatives, especially at the larger sample sizes. However, the robustness for these two estimators breaks down in case of a weak overall factor structure, in which the idiosyncratic component explains a larger part of the variability than the common factors together, i.e. in case the inverse signal to noise ratio $\theta > 1$. The fifth panel in Table 1 and Table 2 considers configurations of an overall weak factor structure. The results confirm the outperformance of the ED and DJS estimators.

Comparison of ED and DJS: impact of weak factor

Figure 3 illustrates the impact of the introduction of a weak factor for the ED and DJS estimators. While the upper right panel of Figure 2 plots the the frequency of incorrect number of estimated factors for $\theta_{wf} = \theta = 1$,. Figure 3 plots this statistic for all values of θ_{wf} and $\theta = 2$. The mode of the estimated number of factors being correct, i.e. $r = 3$, is represented by the blue coloured surface, while the opposite is presented by the yellow coloured surface. The ED and DJS estimators converge for large sample sizes in case $\theta_{wf} > 0.75$, while the DJS estimator shows some outperformance for the correct number of factors even for small sample sizes.

Figure 3: Performance of ED and DJS estimators for different factor structures ($\theta = 2$)



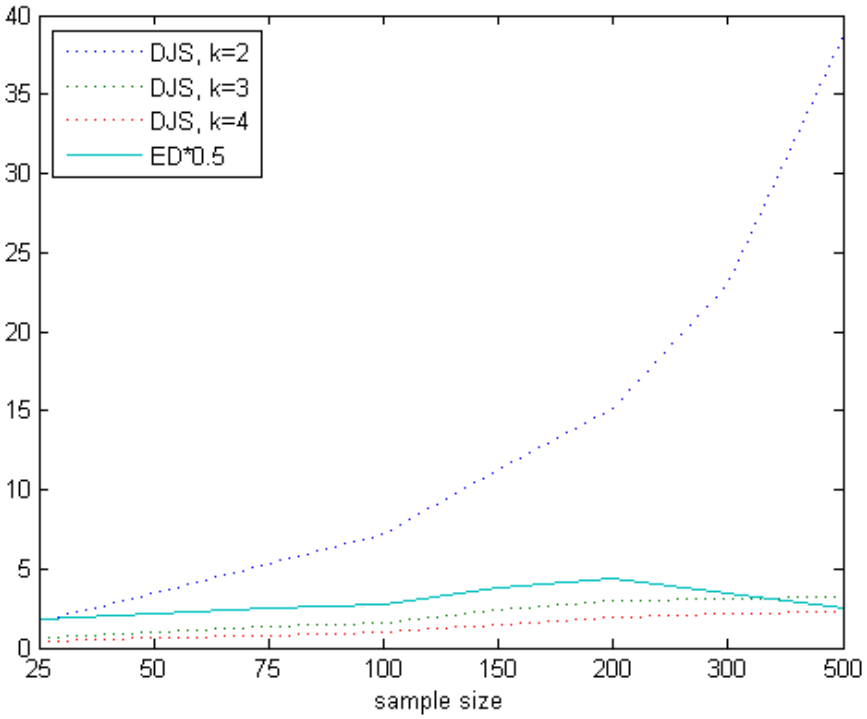
Notes.

The blue coloured surface shows the parameter configurations where the mode of the estimated number of factors is correct, i.e. $r = 3$, while the yellow coloured surface shows the opposite case.

Comparison of ED and DJS: size of threshold

We have seen above in Section 2 that our DJS estimator belongs to the family of estimators of Onatski (2010), the only difference with the estimator ED being the value of the threshold $\hat{\delta}_{k+1}$ in Equation (4). Figure 4 shows the difference between the thresholds of the two estimators for different sample sizes for simulations with $r = 3$ factors for the case $\theta = \theta_{wf} = 1$.

Figure 4: Comparison of ED and DJS thresholds ($\theta = 1$ and $\theta_{wf} = 1$, and $r = 3$ factors)



The graph shows the ED threshold multiplied by .5, since Onatski determines the threshold with a regression and multiplies the outcome ad hoc by 2. Our DJS threshold is equal to $\hat{\lambda}_{k+1}/k$ and shown for $k = 2,\ 3$ and 4 where λ are the scaled eigenvalues of $\boldsymbol{X}'\boldsymbol{X}/T$. The figure shows that the

threshold $\hat{\delta}_3$ which corresponds to $k = 2$ diverges for large sample sizes as expected whereas $\hat{\delta}_4$ and $\hat{\delta}_5$ which correspond to $k = 3$ and $k = 4$ respectively, converge and are close to the ED threshold.

4 Conclusion

This paper presents a simple criterion to determine the number of factors in a data-rich environment, based on the comparison of surfaces under the scree plot. Our procedure is intuitive appealing and straightforward to implement. Our procedure is closely related to Onatski (2010), but is more straightforward (and therefore more efficient). Monte Carlo simulations taking into account weak factors reveal that our criterion outperforms the two eigenvalue test ratios of Ahn and Horenstein (2013) for all sample sizes, and Onatski's (2010) edge distribution estimator, except for large samples.

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